VII. *Determination of the Heliocentric Longitude of the descending Node of* Saturn. *By* Thomas Bugge, *Professor of Astronomy in the University of* Copenhagen; *communicated by Sir* Joseph Banks, *Bart. P. R. S.*

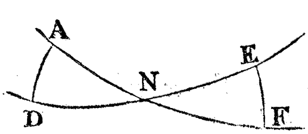
Read December 7, 1786.

THE culmination of Saturn was observed with a 6-feet achromatic transit-instrument, and the planet compared with *o* and *π* of Sagittarius, whose apparent right-ascensions in the middle of August 1784 were 282° 56′ 54″ and 284° 14′ 33″. The meridian altitude was observed with a 6-feet mural quadrant. The original observations are to be published in the second volume of my Astronomical Observations. From those are calculated the right-ascension and declination, the geocentric longitude and latitude, of Saturn, which are to be depended upon to 4 or 6 seconds. Those observed longitudes and latitudes are compared with the tables of Dr. HALLEY and of M. DE LA LANDE. In the errors of the tables + signifies that the longitude of the tables is less than the observed longitude; and the meaning of − is, that the calculated longitude is greater than the observed. It ought to be observed, that the heliocentric longitudes of Dr. HALLEY'S Tables have been corrected for the perturbations after the principles of M. LAMBERT. (Memoires de Berlin, pour 1783, p. 216. and Collection des Tables Astronomiques de Berlin, tom. II. p. 269.)

1784	♄ culmination, mean time at Copen.	♄ observed longitude.	♄ observed latitude.	The error of HALLEY in long.	The error of HALLEY in lat.	The error of M. DE LA LANDE in long.	The error of M. DE LA LANDE in lat.
	h. ′ ″	s. ° ′ ″	° ′ ″	′ ″	″	′ ″	″
July 12	12 3 1	9 20 34 48	0 3 35 B	+2 22	+32	−9 40	+33
20	11 29 9	9 19 59 39	0 2 59	+2 14	+38		
Aug. 1	10 38 25	9 19 9 22	0 1 44	+1 45	+27	−9 49	+28
8	10 9 0	9 18 42 56	0 1 2	+1 48	+21		
21	9 14 59	9 18 1 23	0 0 2	+1 35	+28		
27	8 50 19	9 17 46 19	0 0 29A	+1 26	−26	−9 35	−30
31	8 33 47	9 17 38 7	0 0 53	+1 19	−23		
Sept. 5	8 13 45	9 17 29 36	0 1 26	+1 20	−25	−9 25	−27
15	7 33 45	9 17 19 39	0 2 4	+1 18	−25		
Oct. 8	6 4 23	9 17 34 6	0 3 57	+1 22	−26	−8 48	−32

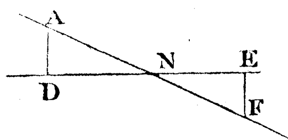
In order to reduce the obſerved geocentric longitude to the ſun, or by obſervation to find the heliocentric longitude, it is required to know the angle at the planet $=p$. If this angle is calculated in the common way only by the tables, there will ariſe ſome difference, according to the different elements and the different conſtructions of the tables. Thus, at the time of Saturn's culmination, this angle is found the 12th of July, by the tables of Dr. HALLEY $=0° 3' 13''$, and by the tables of M. DE LA LANDE $=0° 2' 0''$; the 8th of Auguſt by HALLEY $=2° 43' 35''$, and by M. DE LA LANDE $=2° 42' 34''$; the 27th of Auguſt after Dr. HALLEY $=4° 14' 15''$, and after M. DE LA LANDE $=4° 13' 47''$. To avoid thoſe differences, which often may alter the heliocentric longitude more than one or two minutes, the following method may be uſeful. The heliocentric longitude of the earth, calculated after the tables of M. MAYER, is to be depended upon to eight or ten ſeconds. From the heliocentric longitude of the earth, and from the obſerved geocentric longitude of the planet, corrected for the aberration and nutation, is deduced the angle at the earth $=t$, or the diſtance between the ſun and the planet ſeen from the earth. The dimenſions of the elliptical orbit of the planet are ſo far aſcertained, that the logarithms of the diſtance from the ſun have not any material difference in the different tables. From the angle t, the diſtance of the earth from the ſun, and the diſtance of the planet from the ſun, the angle p is calculated to a ſufficient degree of accuracy. Thus, the 12th of July, by the diſtances of Dr. HALLEY, $p = 0° 2' 59''$, and by the diſtances of M. DE LA LANDE $= 0° 2' 59''$; the 8th of Auguſt after Dr. HALLEY $p = 2° 43' 25''$, and after M. DE LA LANDE $p = 2° 43' 36''$; the 27th of Auguſt after Dr. HALLEY $p = 4° 14' 10''$, and after M. DE LA LANDE $p = 4° 14' 28''$. The difference very ſeldom will amount to 20 ſeconds, and is of no conſequence in this matter. From the obſerved geocentric latitude of the angle at the ſun $= s$, and the angle at the earth $= t$, the heliocentric latitude of the planet is found $= \frac{\text{tang. lat. geoc.} \times \text{ſin. } s}{\text{ſin. } t}$.

1784	Mean time at Copenhagen.	♄ obſerved heliocentric longitude.	♄ obſerved heliocentric latitude.
	h. ′ ″	s. ° ′ ″	° ′ ″
July 12	12 3 1	9 20 37 29	0 3 13 B
20	11 29 9	9 20 51 53	0 2 41
Aug. 1	10 38 85	9 21 13 17	0 1 34
8	10 9 0	9 21 26 2	0 0 56
21	9 14 59	9 21 49 27	0 0 2
27	8 50 19	9 22 0 12	0 0 27 A
31	8 33 47	9 22 7 32	0 0 50
Sept. 5	8 13 45	9 22 16 28	0 1 21
15	7 33 45	9 22 34 32	0 1 59
Oct. 8	6 4 23	9 23 16 15	0 3 35

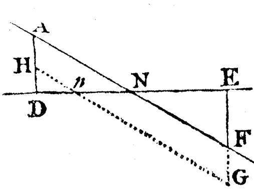


When two heliocentric longitudes, and the correſponding northern and ſouthern latitude are given, the diſtance of of the node from one of the longitudes or places may be found. Let DE be the ecliptic, AF the orbit of the planet, N the node, DE the difference between the two obſerved heliocentric longitudes $=a$, EF the ſouthern latitude $=\beta$, AD the northern latitude $=b$, NE the diſtance of the node from the heliocentric place at E, and correſponding to the ſouthern latitude $=x$. In the ſpherical triangles ADN and FEN, $\frac{\text{ſin.}\,(a-x)}{\text{tang.}\,b}=\text{cot. N}=\frac{\text{ſin.}\,x}{\text{tang.}\,\beta}$. By placing the value of ſin. $(a-x)$ in the equation $\frac{\text{ſin.}\,a\,.\,\text{coſ.}\,x-\text{ſin.}\,x\,.\,\text{coſ.}\,a}{\text{tang.}\,b}=\frac{\text{ſin.}\,x}{\text{tang.}\,\beta}$. By reſolving this equation $\frac{\text{ſin.}\,x}{\text{coſ.}\,x}=\text{tang.}\,x=\frac{\text{ſin.}\,a\,.\,\text{tang.}\,\beta}{\text{tang.}\,b+\text{coſ.}\,a\,.\,\text{tang.}\,\beta}$.

If a, b, and β, are very ſmall arcs, which commonly is the caſe with the planets, then ſin. $a=a$, tang. $\beta=\beta$, tang. $b=b$, and coſ. $a=1$. Hence the ſpherical formula will be transformed into another $x=\frac{a\beta}{b+\beta}$. This formula belongs to plane geometry, and may beſides be thus demonſtrated. DN : NE



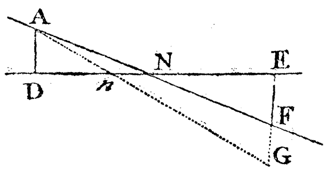
$=$ AD : EF. Hence DN + NE : NE $=$ AD + EF : EF; and NE $=\frac{\text{DE}\times\text{EF}}{\text{AD}+\text{EF}}$. If the difference of the longitudes do not exceed one degree, and the latitudes are not greater than ten minutes, the ſpherical and the rectilineal formula will agree to very few ſeconds. Small faults in the longitude will not very much alter the true place of the node; but very ſmall errors in the latitude are of great conſequence. Let the error in the



ſouthern heliocentric latitude be FG $=+d$. The error in the northern latitude AH $=-d$. Hence DH : Dn $=$ GE : En, and E$n=\frac{a\,(\beta+d)}{b+\beta}$. By ſubtracting EN

=

$=\frac{a\beta}{b+\beta}$, the error in the heliocentric longitude of the node, $Nn=\frac{ad}{b+\beta}$. If the fault in the ſouthern latitude $= -d$, in the northern latitude $= +d$, the ſame formula is ſtill true; but then $EN > En$, and the place of the erroneous node will be between E and N. In both caſes the errors in the place of the node are directly as the errors in the latitudes.



Let us now ſuppoſe, that only the one latitude is erroneous $\pm d$. Then $Nn = \pm En \mp EN = a \times \left(\pm\frac{\beta\pm d}{b+\beta\pm d} \mp \frac{\beta}{b+\beta}\right) = \frac{abd}{(b+\beta)^2 \pm d(b+\beta)}$. In the caſe when the error in both latitudes is poſitive $= +d$, and $\beta > b$, or $\beta < b$, the reſulting error in the place of the node $= \frac{ad(\mp b\pm\beta)}{(b+\beta)^2+2d(b+\beta)}$. In the caſe when the error in both latitudes is negative $-d$, and $\beta > b$, or $\beta < b$, then the error in the node $= \frac{ad(\mp b\pm\beta)}{(b+\beta)^2-2d(b+\beta)}$. In thoſe two caſes the error is leſs than in any of the former, and quite nothing when $b=\beta$. If the radius of the inſtrument, with which the meridian altitudes are obſerved, is given, the quantity of d is alſo given. In a mural quadrant of 6 or 8 feet $d = 5$ or 3 ſeconds. Take $a = 34'\ 3''$, $b = 56''$, $\beta = 27''$, $d = 5''$, and the error in the ſouthern latitude $+ d$, in the northern $= -d$; then $Nn = \frac{10215''}{83} = 2'\ 12''$. Take now the error only in the ſouthern latitude $= +d$; then $Nn = \frac{572040''}{7304} = 1'\ 18''$; in the caſe of $-d$; $Nn = \frac{572040''}{6474} = 1'\ 28''$. From hence it appears, that in comparing

 two

two ſingle obſervations, it ſcarce will be poſſible to avoid a fault of ± 2 minutes in the place of the node.

If the inſtrument is of a leſs force than a mural quadrant of 6 feet, and the poſſible faults in the altitudes greater, for example, 10 or 15 ſeconds, the reſulting error in the place of the node may very eaſily be calculated; but the error in the node will be enormous, and the obſervations of no uſe for a nice aſtronomer.

Compared Obſervations 1784.	♄ heliocentric longitude on the laſt day.	♄ diſtance from the ☋.	Heliocentric longitude of ♄ ☋
	s. ° ′ ″	° ′ ″	s. ° ′ ″
July 12 with Sept. 15	9 22 34 32	0 44 38	9 21 49 54
July 12 — Oct. 8	9 23 16 15	1 23 41	9 21 52 36
July 20 — Sept. 15	9 22 34 32	0 43 37	9 21 50 55
July 20 — Oct. 8	9 23 16 15	1 22 33	9 21 53 42
Aug. 1 — Sept. 5	9 22 16 28	0 29 15	9 21 47 13
Aug. 1 — Sept. 15	9 22 34 32	0 45 23	9 21 49 9
Aug. 1 — Oct. 8	9 23 16 15	1 25 34	9 21 50 41
Aug. 8 — Aug. 27	9 22 0 12	0 11 7	9 21 49 5
Aug. 8 — Aug. 31	9 22 7 32	0 19 34	9 21 47 58
Aug. 21 — Aug. 27	9 22 0 12	0 10 0	9 21 50 12
Aug. 21 — Aug. 31	9 22 7 32	0 17 23	9 21 50 9
		Mean	9 21 50 8,5

This mean agrees pretty well with the obſervations on the 21ſt, 27th, and 31ſt of Auguſt, which are neareſt the node, and moſt to be depended upon.

The 21ſt of Auguſt, at 9 h. 12′ 26″ true time at Copenhagen, the heliocentric longitude of Saturn = 9 s. 21° 49′ 27″, and the diſtance from the node = 41″. The 27th of Auguſt, at 8 h. 49′ 23″, the heliocentric longitude = 9 s. 22° 0′ 12″; therefore, in 5 days 23 h. 36′ 57″ Saturn has deſcribed an arc of 10′ 45″, and 10′ 45″ : 5 d. 23 h. 36′ 57″ = 41″ : *x*. Hence

 Saturn

Saturn has spent 9 h. 7′ 44″ in going through those 41″; and *Saturn's passage through the node happened August* 21, 1784, *at* 18 h. 20′ 10″, *and the heliocentric longitude of his descending node* = 9 s. 21° 50′ 8″,5. The errors in the place of the node are relative to the tables of Dr. HALLEY + 19′ 39″, to the tables of M. CASINI + 16′ 4″, and to the tables of M. DE LA LANDE + 1′ 31″.

In the foregoing computation of Saturn's heliocentric longitude from the tables of Dr. HALLEY, this longitude has been corrected for the perturbation after the principles of M. LAMBERT. Though the geocentric places, calculated in this manner, will agree still better with the observations than without those perturbations, nevertheless they are only empiric, and not founded upon the theory and principles of gravitation; I shall therefore conclude this Paper, by adding the faults in the heliocentric places of Saturn, calculated only and directly from the tables of Dr. HALLEY, which may be of some use to improve those valuable tables.

1784.	♄ heliocentric longitude from Dr. HALLEY's tables.	Error in longitude.	♄ heliocentric latitude from Dr. HALLEY's tables.	Error in latitude.
	s. ° ′ ″	′ ″	° ′ ″	″
July 12	9 20 27 40	+9 49	0 2 45 B	+28
20	9 20 42 3	+9 50	0 2 17	+24
Aug. 1	9 21 3 41	+9 36	0 1 10	+24
8	9 21 16 19	+9 43	0 0 37	+21
21	9 21 39 45	+9 42	0 0 24 A	−22
27	9 21 50 34	+9 38	0 0 53	−26
31	9 21 57 47	+9 45	0 1 11	−21
Sept. 5	9 22 6 48	+9 40	0 1 45	−24
15	9 22 24 50	+9 42	0 2 22	−23
Oct. 8	9 23 6 33	+9 44	0 4 1	−26